On the Storage of Heat in Building Components*

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Abstract

Examining heat storage in building construction materials and assemblies — often referred to as thermal storage — by means of a periodically oscillating model of thermal conduction leads to a generalized conductance concept (complex conductance matrices). This concept can serve as the basis for three- dimensional, non-steady-state calculations of the thermal performance of buildings. Relationships to the concept of effective thermal capacity (previously introduced in other publications) are established in context.

1 Introduction

The thermal capacity of building components is of particular importance to the thermal performance of enclosed spaces under summer conditions. Although this fact has been generally acknowledged for some time, its ramifications still have not been entirely researched and determined.

Early attempts at quantifying heat storage capacities can be found in *Bruckmayer*'s work [1], which introduces the concepts of "equivalent-storing brick thickness" and "cooling time value". Later (1952), *Sklover* [2] introduces "damping" and "phase shifts" of building components — terms which have also found their way into standards for building performance in the former Democratic Republic of Germany. For the first time, such values account for periodic fluctuations (as opposed to *Bruckmayer*'s values), with a period lasting 24 hours. Towards the end of the sixties, *Heindl* [3] published values for "temperature amplitude dampening", "thermal alternate flow resistance", etc. Because of poorly chosen boundary conditions, however, these values proved to be of little practical use.

Around 1970, on the basis of building component matrices (already mentioned by *Carslaw–Jaeger* [5] and applied to the study of building thermal performance by *Heindl* [3]), *Haferland*, *Heindl*, and *Fuchs* [4] developed a computer program which allows the calculation of daily internal temperature fluctuations while taking into account the thermal capacity of building components. Here, only one-dimensional heat flow patterns have been considered.

The concept of effective thermal capacity of plane building components composed of several homogeneous layers, as later introduced by *Heindl* [6], actually represents a theoretical regression in comparison to the computer-aided model. Nonetheless, since it apparently better suits the practical need for easily comprehensible values, Heindl's simplified model has even found its way into Austrian building standards (specifically B8110). This is not identical, however, to the concept introduced here for "effective thermal capacity". Furthermore, limiting its application to plane (slab-shaped) components is unnecessary — as shall be shown in the following.

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It would certainly not be relevant to consider heat storage in an isolated manner, independent of other thermal problems. In order to more completely, systematically describe this subject, a larger framework must be introduced.

2 The thermal conduction model

Heat flow in a building occurs by means of conduction, convection, and radiation. Within a building component, conduction plays the primary role; convection and radiation are generally secondary.

For our purposes, we can visualize a solid body of material with one exterior surface (in contact with outside air) and one or more interior surfaces (bordering various inner spaces). The diagram below, illustration 1 — two-dimensional for the purpose of simplification — should illustrate this.

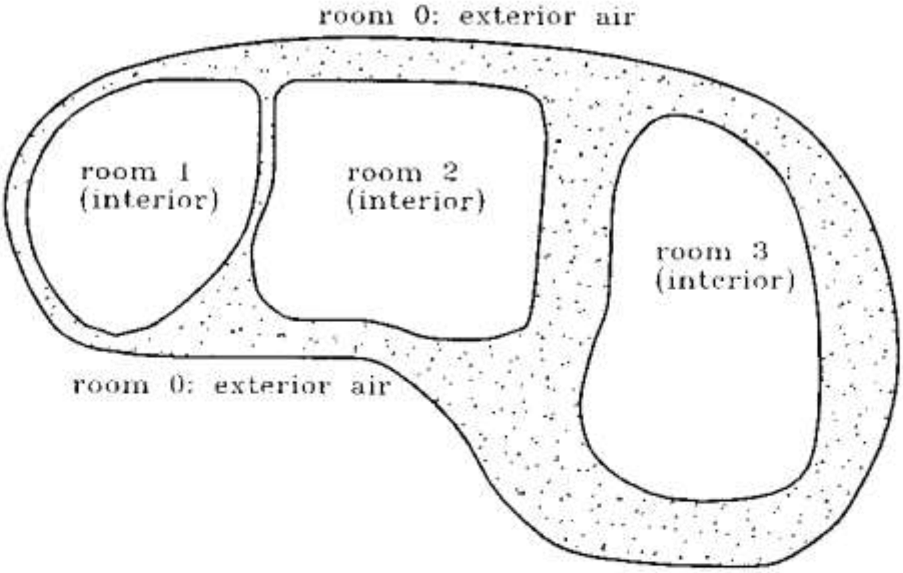

Figure 1: diagram of a building with three interior spaces

Though the form of representation might appear peculiar to an architect, it has been chosen to reduce the significance of the generally common — but for our purposes unnecessary — distinction between interior and exterior walls.

As is well known, heat transmission in a conducting solid is described by Fourier's law of heat conduction, which relates the heat flow rate $\vec{q}$ to the gradient of temperature U:

$$\vec{q} = -\lambda \cdot \mathrm{grad} U \quad . \tag{1}$$

For the isotropic case, the thermal conductivity is a scalar; in the event of anisotropy (for example, with the building material wood), it is a second-order symmetric tensor.

Fourier's law of heat conduction has yet to be completed by a heat balancing equation which connects the field source of the heat flow rate $\vec{q}$ with the time-dependancy of temperature U:

$$c \cdot \varrho \cdot \frac{\partial U}{\partial t} = -\mathrm{div}(\vec{q}) \quad , \tag{2}$$

in which c is the specific heat and ϱ the mass per unit volume of a given building material. This equation is only valid for the case that no heat source is located within the solid, the only case that is of interest to us here.

For the following, it shall be assumed that λ, c and ϱ are all strictly functions of position, in particular, independent of temperature U. If we work Fourier's law (1) into equation (2), we obtain the familiar equation of heat flow in the form:

$$c{\cdot}\varrho{\cdot}\frac{\partial U}{\partial t} = \mathrm{div}(\lambda{\cdot}\mathrm{grad}U) \quad . \tag{3}$$

This differential equation is linear and homogeneous, and can furthermore be satisfied by any constant.

The heat conduction equation has a myriad of solutions, of which a specific solution can only be determined by first choosing further conditions. Particularly the boundary conditions must be defined, i.e. the conditions which must be satisfied along the boundaries of the assumed region.

With respect to the particular application, it is reasonable to assume so called boundary conditions of the third kind for the surfaces of the interior spaces, which ultimately amounts to defining an interior air temperature for each space. For most cases, this thermal coupling of the interior surface with the interior air temperature represents a reasonably precise approximation — not an exact description — of the actual circumstances. It takes the exchange of long-wave radiation in the space into consideration only very indirectly, but still proves to be effectively the only possibility for keeping the practical scope of calculations necessary to arrive at a solution within justifiable limits. Also, by taking too many details into consideration, it would become nearly impossible to maintain a coherent picture of the essential physical relations, criteria for rational design.

The boundary conditions for the exterior of the building differ from those for the interior solely in that outside, the "air temperature" may also depend on location. This proves to be necessary because the influence of radiation — in particular solar — can by no means be neglected on the exterior surface. It is well known that allowances can be made for such radiation effects within the definition of an "air temperature" [7], [8], [9]. The spatial dependancy of the exterior temperature will usually be accounted for by introducing several "exterior spaces" — each with an air temperature that is the same for all points withing the space. Such a procedure also proves useful for building components in contact with the ground. Of course, the analysis of wandering shadow boundaries is not possible with this method; this shall be left to analyses based on other concepts.

Taking the heat transfer coefficient α_k into consideration, the given air space temperatures $T_k, k = 0, 1, \ldots, m$, lead to boundary conditions of the third kind — see [10], page 75. For the boundary surface $\mathcal{R}_k$, i.e. the portion of the building surface in contact with the air of the k–th space, this condition reads:

$$(\lambda{\cdot}\mathrm{grad}U){\cdot}\vec{n} + \alpha_k{\cdot}U = \alpha_k{\cdot}T_k \quad . \tag{4}$$

Here $\vec{n}$ signifies the unit vector perpendicular to the building component surface and directed into the k–th space.

Boundary condition (4) must be stated for each space concerned. If not all boundaries of the building structure are thus covered, then further boundary conditions must be applied. Should it be required that no heat pass through any such artificially defined boundaries, then one finds boundary conditions of the second kind (see [10], page 75):

$$(\lambda\cdot\text{grad}U)\cdot\vec{n} = 0 \qquad . \tag{5}$$

Since this is just a special case for the third kind, we can continue under the assumption that conditions of the third kind are given along all boundaries.

The imposition of boundary conditions alone is not sufficient to uniquely determine a solution for the heat conduction equation (3). In addition, we must either define an initial condition, i.e. prescribe the temperature distribution in the structure for a specific initial point in time, or impose a different suitable condition.

Initial conditions are of no interest in the context discussed here. Time–independence, where feasible, is often required instead, in which case the left side of the heat conduction equation (3) disappears. This case is treated in [11]. When examing heat storage in buildings, one cannot, of course, limit the analysis to this steady–state case. The relevant effects of temporary heat storage in building assemblies only come to light when the daily rise and fall of the exterior air temperature as well as interior temperature fluctuations (for the most part, also of a daily rhythm) are taken into consideration. This suggests envisaging the periodicity of all the continuos processes rather than imposing initial conditions.

A temperature function which is bounded and periodic in time can always be written as a Fourier series, that is,

$$U(x,y,z,t) = \frac{1}{2}\cdot\sum_{\nu=-\infty}^{+\infty} u_\nu(x,y,z)\cdot e^{i\cdot\frac{\nu\cdot\pi\cdot t}{l}} \qquad , \tag{6}$$

with $2l$ denoting the time period of the function. If we take this approach into the differential equation (3) and collect terms, we obtain the following differential equation for the Fourier coefficient $u_\nu(x,y,z)$:

$$i\cdot c\cdot\varrho\cdot\frac{\nu\cdot\pi}{l}\cdot u_\nu = \text{div}(\lambda\cdot\text{grad}u_\nu) \qquad . \tag{7}$$

With the abbreviation $\frac{\nu\cdot\pi}{l} = \omega$ and by dropping the index ν (unnecessary for comprehensibility here), this becomes

$$i\cdot c\cdot\varrho\cdot\omega\cdot u = \text{div}(\lambda\cdot\text{grad}u) \qquad , \tag{8}$$

a differential equation for the Fourier coefficient u, in which time is naturally no longer a variable.

The Fourier coefficients u of the temperature U are easily graphically interpreted. As the frequency of rotation approaches zero, the function approaches the steady–state case. For $\omega = 0$, u becomes the mean value over time of temperature U — for each considered point respectively. For the value of frequency ω related to the period of a day (as for any value of ω not equal to zero), we must expect a complex function rather than a real value solution to the differential equation (8). The absolute value of the complex number u supplies the (real) amplitude of the sinus curve run of temperature U; the argument supplies the phase information. In the following, we shall also refer to u as the "complex amplitude" of the harmonic related to ω of the temperature function. This, of course, has the dimension of temperature.

A special solution to the differential equation (8) can be uniquely determined by imposing boundary conditions; initial conditions are not required since time is not a variable in (8). The boundary conditions for the differential equation (8) result from carrying over the conditions formulated for a time–dependant heat conduction equation (4) into the Fourier coefficients:

$$(\lambda\cdot\text{grad}u)\cdot\vec{n} + \alpha_k\cdot u = \alpha_k\cdot t_k \qquad . \tag{9}$$

Solving a boundary value problem of this kind in a self-contained manner is only possible for a few simple cases. Generally, numeric methods aided by electronic data processing are required. One tries to avoid repeating such an investment when new boundary values — in our case, new Fourier coefficients for air temperatures — should be considered.

A reduction of the effort involved becomes possible because the linearity and homogeneity of differential equation (8) allow the superposition of solutions which always lead back to solutions of the differential equation. Accordingly, it is sufficient to calculate a number of "basic solutions" which can then be combined to fulfill actual boundary conditions.

A basic solution $g_j(x,y,z)$ is defined by prescribing the amplitude t_j with a value of 1 in the space j and with a value of 0 in all other spaces. Hence the basic solutions are determined by the boundary condition

$$(\lambda \cdot \text{grad} g_j) \cdot \vec{n} + \alpha_k \cdot g_j = \alpha_k \cdot \delta_{kj} \tag{10}$$

for $j = 0, 1, \ldots m$ along all boundaries $\mathcal{R}_k, k = 0, 1, \ldots, m$. The *Einstein* summation convention is not applicable to this equation; δ_{kj} stands for the Kronecker delta.

By linear combination of basic solutions, the expression $u(x,y,z)$ of the differential equation (8) for given Fourier coefficients t_j then becomes

$$u(x,y,z) = \sum_{j=0}^{m} t_j \cdot g_j(x,y,z) \quad . \tag{11}$$

The basic solutions are dimensionless by definition. The Fourier coefficient u receives the dimension of a temperature through the Fourier coefficients t_j of the air temperatures.

Given the calculation of the Fourier coefficients $u(x,y,z)$, which must, of course, be performed for all required harmonics, the expression of the temperature function $U(x,y,z)$ in space and time is no longer difficult: we simply substitute them into equation (6) and carry through the summation.

For practical use, essentially just one daily and the seasonal periods are of importance (the seasonal especially in connection with building components in contact with the ground). In some cases, it may also make sense to consider a seven day period, particularly when the interior temperature fluctuations correlate to weekly periodic occupant habits.

3 The conductance concept

The calculation of the temperature distribution in a building structure is not an end in itself. Though temperatures occurring on the surface of and within building components are of interest, particularly in connection with moisture problems, a different question is frequently of primary importance: what heat flow occurs through the boundaries of a space when the air of this space is subjected to a given temperature or to a defined temperature fluctuation? The evident goal here is ultimately the question of what heating load is required to maintain a desired temperature in a particular space.

The heat flow Q_i, which flows out of space i through boundary $\mathcal{R}_i$, is apparently given by

$$Q_i = - \iint_{\mathcal{R}_i} (\lambda \cdot \text{grad} U) \cdot d\vec{a} \quad , \tag{12}$$

in which $d\vec{a}$ is the surface element oriented from the space toward the building component. The Fourier coefficient q_i for the heat flow Q_i is accordingly

$$q_i = -\iint_{\mathcal{R}_i} (\lambda \cdot \text{grad} u) \cdot d\vec{a} \quad . \tag{13}$$

If we implement the expression from equation (11) for the Fourier coefficient $u(x,y,z)$, then we obtain

$$q_i = -\sum_j t_j \cdot \iint_{\mathcal{R}_i} (\lambda \cdot \text{grad} g_j) \cdot d\vec{a} \quad . \tag{14}$$

We shall designate the value of the surface integral of the j-th basic solution g_j taken over the boundary $\mathcal{R}_i$ of the i-th space as L_{ij}:

$$L_{ij} = \iint_{\mathcal{R}_i} (\lambda \cdot \text{grad} g_j) \cdot d\vec{a} \quad . \tag{15}$$

The quantity L_{ij} has the dimension of thermal conductance. From now on we shall refer to it as the "generalized conductance". When the frequency $\omega = 0$ (i.e. for the steady-state condition or the 0-th harmonic), L_{ij} reduces to the familiar real steady-state thermal conductance value — see [11], [12]. When $\omega \neq 0$, L_{ij} becomes complex in value.

Now equation (14) takes the form

$$q_i = -\sum_j L_{ij} \cdot t_j \quad . \tag{16}$$

If we allow the i here to run from 0 to m, then we obtain a system of equations which combines the complex air temperature amplitudes t_j with the complex heating load amplitudes q_i. The thermal capacity of the air in the course of heating and cooling has been assumed negligable, thus allowing the interpretation of q_i as the heating load. In this case, it is evident that heat flowing out through the space boundaries must correspond to heat supplied to the air within the space at every point in time.

The significance of L_{ij} for steady-state conditions, i.e. $\omega = 0$, is discussed in detail in [11]. For $\omega \neq 0$, the matrix of the generalized conductances L_{ij} must first be proven symmetric; this follows in a form analogous to the proof of symmetry for the symmetric steady-state conductance matrix in [11] and can therefore be kept brief here.

The definition equation (15) of the generalized conductance can also be written as

$$L_{ij} = \sum_k \iint_{\mathcal{R}_k} \delta_{ki} \cdot (\lambda \cdot \text{grad} g_j) \cdot d\vec{a} \quad . \tag{17}$$

Boundary condition (10) leads to the description of the Kronecker delta as

$$\delta_{ki} = g_i + \frac{1}{\alpha_k} \cdot (\lambda \cdot \text{grad} g_i) \cdot \vec{n} \quad . \tag{18}$$

By inserting this into (17), it follows that

$$L_{ij} = \sum_k \iint_{\mathcal{R}_k} g_i \cdot (\lambda \cdot \text{grad} g_j) \cdot d\vec{a} + \sum_k \iint_{\mathcal{R}_k} \frac{1}{\alpha_k} \cdot (\lambda \cdot \text{grad} g_i) \cdot \vec{n} \cdot (\lambda \cdot \text{grad} g_j) \cdot d\vec{a} \quad . \tag{19}$$

The first summation represents a surface integral taken over the entire boundary surface $\mathcal{R}$. If we implement $d\vec{a} = -\vec{n}\,da$[1)] in the second summation, we then obtain

$$L_{ij} = \oiint_{\mathcal{R}} g_i \cdot (\lambda \cdot \mathrm{grad} g_j) \cdot d\vec{a} - \sum_k \iint_{\mathcal{R}_k} \frac{1}{\alpha_k} \cdot [(\lambda \cdot \mathrm{grad} g_i) \cdot \vec{n}] \cdot [(\lambda \cdot \mathrm{grad} g_j) \cdot \vec{n}] \cdot da \quad . \tag{20}$$

The integral after the summation symbol is symmetrical in i and j. The surface integral can be transformed according to *Gauss* into

$$\oiint_{\mathcal{R}} g_i \cdot (\lambda \cdot \mathrm{grad} g_j) \cdot d\vec{a} = - \iiint_{\mathcal{G}} [\mathrm{grad} g_i \cdot (\lambda \cdot \mathrm{grad} g_j) + g_i \cdot \mathrm{div}(\lambda \cdot \mathrm{grad} g_j)] \cdot dG \quad . \tag{21}$$

Upon applying differential equation (8), which must be fulfilled for the basic solution g_j, this becomes

$$\oiint_{\mathcal{R}} g_i \cdot (\lambda \cdot \mathrm{grad} g_j) \cdot d\vec{a} = - \iiint_{\mathcal{G}} [\mathrm{grad} g_i \cdot (\lambda \cdot \mathrm{grad} g_j) + i \cdot c \cdot \varrho \cdot \omega \cdot g_i \cdot g_j] dG \quad . \tag{22}$$

This integral is also evidently symmetric in i and j. Hence it is also valid for the generalized conductance that

$$L_{ij} = L_{ji} \quad . \tag{23}$$

The practical significance of the generalized conductance values can be inferred from equation (16), which establishes a relationship between the amplitudes t_j of the respective air temperatures on the one side and the amplitudes q_i of the heating loads in the respective spaces on the other. The real conductance values for the steady–state condition do the same for the mean values of temperature and heating load — see [11].

Thus the system of equations (16) — to be presumed applicable for mean values (steady-state) as well as for every relevant harmonic — is the fundamental set of equations for the calculation of heating loads. It is even suited for a time–dependant ("dynamic") evaluation of heating loads, something not generally considered in building performance standards. It should be noted that the system of equations (16) is equally suited for calculating air temperatures (mean values and complex amplitudes) with given or partially given heating loads. In particular, it represents the essential foundation for calculating summer interior temperature fluctuations within a building. The fact that q_i represents heating load amplitudes (and mean values) of the interior air but not of the surfaces and materials of building components is insignificant, as shall be shown in a separate discourse.

4 The effective thermal capacity

Given periodic heating and cooling processes within a given space (typically assumed to be of a daily period), a given heating load amplitude induces slighter or greater temperature fluctuations depending on the type of construction. Conversely: maintaining a given air temperature amplitude requires a relatively small heating load amplitude in a light form of construction and a large one in buildings consisting of thick and heavy masses.

[1)] Here there is a sign error in [11], which nevertheless has no effect on the result.

The thermal capacity of a homogeneous solid of mass m with specific heat c is — as is well-known — given by $m{\cdot}c$, i.e. the increase δE of the heat quantity stored due to a temperature increase δT is

$$\delta E = m{\cdot}c{\cdot}\delta T \quad . \tag{24}$$

This expression is valid during a heating or cooling process for any time interval chosen if we assume that the temperature increases uniformly throughout the entire solid. For this fictitious border case, we can also proceed with time derivatives and write

$$\frac{dE}{dt} = m{\cdot}c{\cdot}\frac{dT}{dt} \quad . \tag{25}$$

The left side represents the heat flowing into the solid, Q:

$$Q = m{\cdot}c{\cdot}\frac{dT}{dt} \quad . \tag{26}$$

Temperature T is that uniform temperature which manifests itself within the solid as well as in its environment. If this temperature fluctuates with an amplitude θ, then

$$T = \theta{\cdot}\sin\omega t \quad . \tag{27}$$

Substituting into (26) delivers

$$Q = m{\cdot}c{\cdot}\theta{\cdot}\omega{\cdot}\cos\omega t \quad , \tag{28}$$

in other words, a heat flow with the amplitude

$$\Gamma = m{\cdot}c{\cdot}\omega{\cdot}\theta \quad . \tag{29}$$

Subsequently, this leads to the expression for the thermal capacity $m{\cdot}c$:

$$m{\cdot}c = \frac{\Gamma}{\omega{\cdot}\theta} \quad . \tag{30}$$

This equation can still be employed as a definition for "effective thermal capacity" Ξ even if the temperature distribution under consideration is not uniform.

Though this interpretation of the thermal capacity of a built space is quite plausible, it is certainly not complete. If we allow the air temperature in a space to fluctuate in a prescribed manner, then the amplitude of the heat flow passing through the space boundaries (i.e. the necessary heating load amplitude) still depends on which boundary conditions are valid for the other spaces, including the "exterior spaces".

Only two of many possibilities shall be considered here:

- The temperature in all spaces except for the one under direct consideration is constant (the temperature amplitudes disappear).
- The temperature fluctuations are the same in all of the spaces; in particular, the temperature amplitudes are equal[2].

[2] Nothing, however, prevents the mean temperature values from differing.

The first case is quite simple to relate. If we are interested in the thermal capacity of space i, then equation (16) — thanks to the disappearance of temperature amplitudes in the other spaces — reduces to

$$q_i = -L_{ii} \cdot t_i \quad . \tag{31}$$

For the effective thermal capacity we are seeking, we thus obtain

$$\Xi_i = \frac{|L_{ii}|}{\omega} \quad . \tag{32}$$

This surprisingly simple result concurrently provides an easily comprehensible interpretation of the primary diagonal in the generalized conductance matrix. However, one can also see that the effective thermal capacity Ξ_i contains less information than the complex generalized conductance L_{ii}; it does not include the phase shift between heat load and temperature functions. Therefore effective thermal capacities cannot generally be taken as a sum, even in the case of parallel thermal circuits (whereas conductances are indeed additive in that case).

The second aforementioned case also draws on equation (16), but this time, the same temperature amplitude is applied in all spaces. These amplitudes shall be denoted as t^* and can be lifted out of the summation. Thus

$$q_i = -t^* \cdot \sum_{j=0}^{m} L_{ij} \quad . \tag{33}$$

For the effective thermal capacity corresponding to these boundary conditions Ξ_i^*, we obtain

$$\Xi_i^* = \frac{|q_i|}{\omega \cdot |t^*|} = \frac{1}{\omega} \cdot \left| \sum_{j=0}^{m} L_{ij} \right| \quad . \tag{34}$$

This effective thermal capacity is therefore determined by the absolute column or row sum for the space i in the conductance matrix.

In practical applications, the lesser of the two thermal capacity values Ξ_i and Ξ_i^* would usually be used as a rough estimation. Of course, there is also the possibility of calculating the absolute values of all the partial sums of the matrix row containing L_{ii} to obtain the smallest value for use. The question of whether or not such a procedure proves viable is moot, since the understanding of effective thermal capacity alone will certainly not suffice for a reasonably precise quantitative assertion. The concept of effective thermal capacity can, however, serve well to illustrate complex generalized conductance.

The concept of effective thermal capacity is not new, although it has not — to my knowledge — been treated in such a generalized manner in other publications. For certain simple cases with one-dimensional heat flow, effective thermal capacities can also be described in equations; in more complicated cases, they must be calculated with the help of simple algorithms (matrix method)[3].

[3] It was left undecided in [6] whether or not surface heat transfer coefficients should be taken into account when calculating the effective thermal capacity of a building component. Here it should be emphasized that a specific decision in this point may strongly affect the calculated values.

5 An example

Possible applications of the concepts presented here, in particular that of the conductance matrix, shall now be demonstrated by considering a simple example. In order to bring the essential properties to the fore, a fictitious building structure shall be assumed as a two-dimensional heat conduction case and numerically calculated. For the purpose of simplicity, the example is stripped of all structural details. This does not mean that more complicated practical applications cannot be carried through with the concepts developed here, aided by the computer programs necessary for such numeric evaluations. The rudimentary nature of the example should serve to coherently illustrate the method in principle.

We consider a building (two-dimensional) consisting of three rooms as depicted in illustration 2.

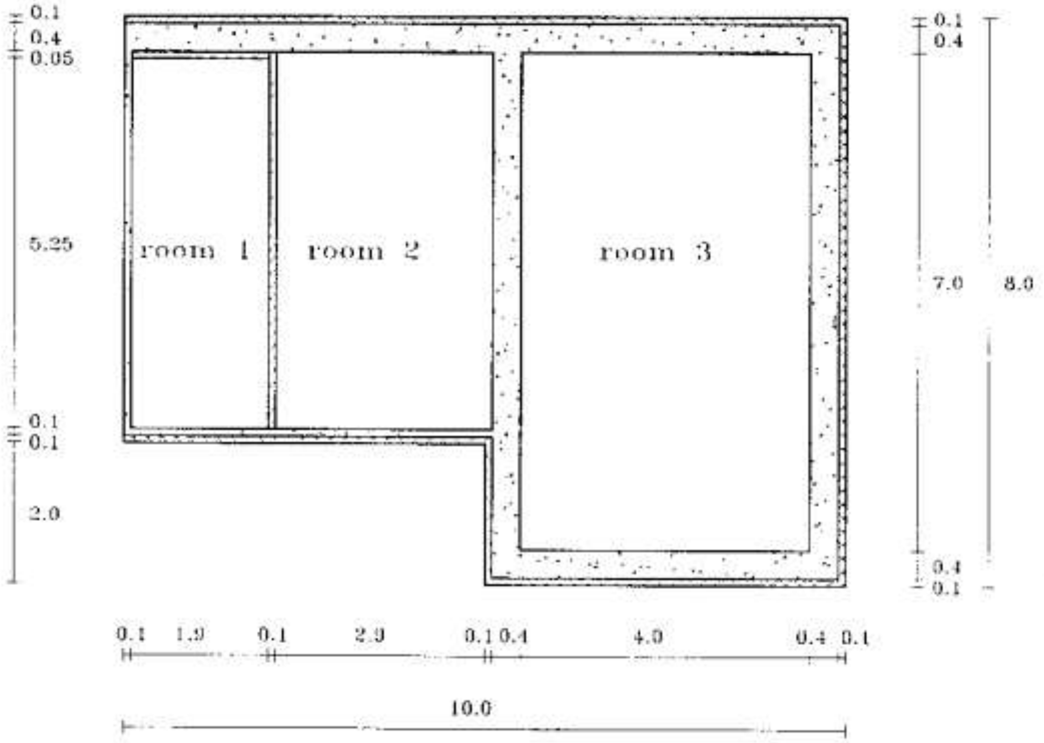

Figure 2: sketch of the assumed building; dimensions in meters.

Except for the partition wall between room 1 and room 2, all walls are of concrete with a mass density of $\varrho = 2300$ kg m^{-3}, a specific heat of $c = 1130$ J kg^{-1}K^{-1}, and a thermal conductivity of $\lambda = 2.3$ Wm^{-1}K^{-1}. The partition wall between room 1 and room 2 is of gypsum wallboard with $\varrho = 600$ kg m^{-3}, $c = 840$ J kg^{-1}K^{-1} and $\lambda = 0.29$ Wm^{-1}K^{-1}. Except for the left wall of room 1, all exterior walls are encased in 0.1 m thick exterior insulation with $\varrho = 20$ kg m^{-3}, $c = 1400$ J kg^{-1}K^{-1} and $\lambda = 0.041$ Wm^{-1}K^{-1}. In addition, the exterior wall segment along the upper edge of room 1 is sheated with a 0.05 m thick layer of insulation of the same material as the exterior. Doors, windows, etc., have been omitted intentionally in this example for the purpose of simplicity. All further dimensions are provided in illustration 2. For the exterior, a surface heat transfer coefficient of $\alpha_e = 20$ Wm^{-2}K^{-1} shall be assumed; for all interior spaces, $\alpha_i = 8$ Wm^{-2}K^{-1}.

Using a personal computer (80486) and a program [13] specifically developed for such applications, all four basic solutions were first evaluated for the steady–state case (mean values) as well as for a 24–hour period. Subsequently, the (length–related) conductance matrix is

$$\mathcal{L}_\infty = \begin{pmatrix} -35.401 & 26.093 & 2.576 & 6.732 \\ 26.093 & -35.749 & 9.656 & 0.000 \\ 2.576 & 9.656 & -26.021 & 13.789 \\ 6.732 & 0.000 & 13.789 & -20.521 \end{pmatrix} , \tag{35}$$

for the matrix of the generalized conductances with a 24–hour period,

$$\mathcal{L}_{24} = \begin{pmatrix} -64.112- & 12.873- & 0.010- & -0.268+ \\ -33.137 \cdot i & -13.827 \cdot i & -0.386 \cdot i & +0.015 \cdot i \\ 12.873- & -53.469- & 7.244- & -0.000- \\ -13.827 \cdot i & -15.908 \cdot i & -3.823 \cdot i & -0.000 \cdot i \\ 0.010- & 7.244- & -79.222- & -1.307- \\ -0.386 \cdot i & -3.823 \cdot i & 21.626 \cdot i & -0.276 \cdot i \\ -0.268+ & -0.000- & -1.307- & -132.035- \\ +0.015 \cdot i & -0.000 \cdot i & -0.276 \cdot i & -26.950 \cdot i \end{pmatrix} . \tag{36}$$

The quantities in the primary diagonal of matrix $\mathcal{L}_\infty$ are irrelevant; they simply guarantee the balance of the column sums to zero, as explained in [11]. From (35) we can deduce that room 1 is the space most strongly coupled with the exterior (room 0) — the length–related conductance is 26.093 $Wm^{-1}K^{-1}$.

The space most weakly coupled with the exterior is room 2 with a length–related conductance of 2.576 $Wm^{-1}K^{-1}$.

The intermediary thermal coupling of the rooms can also be directly deduced from (35). Rooms 1 and 3 are practically not coupled at all; the value of 0.000 $Wm^{-1}K^{-1}$ described in (35) is more precisely $1.175 \cdot 10^{-4} Wm^{-1}K^{-1}$ and therefore beyond the range of decimals shown.

The result from the two–dimensional conductance evaluation between rooms 1 and 2, which are separated only by a gypsum partition wall, is 9.656 $Wm^{-1}K^{-1}$. The U–value of this wall is 1.681 $Wm^{-2}K^{-1}$. If we multiply this with the length of the wall (interior dimension: 5.3 m), we obtain a thermal conductance of just 8.910 $Wm^{-1}K^{-1}$, in other words, a value which is too small — as could be expected.

The conductance between rooms 2 and 3 is shown in (35) to be 13.789 $Wm^{-1}K^{-1}$; the one–dimensionally calculated value here is of an appropriate order of magnitude as well.

The application of the conductance matrix $\mathcal{L}_\infty$ in calculating constant heating loads shall not be treated specifically here; for this, see [14].

The implications of the generalized conductance matrix $\mathcal{L}_{24}$ — equation (36) — cannot be grasped at a glance. This matrix can be discussed somewhat more easily if we first consider only the absolute values of its elements. The matrix of quantities for the generalized conductances with a period of

24 hours in this example emerges as

$$\begin{pmatrix} 72.169 & 18.892 & 0.386 & 0.268 \\ 18.892 & 55.785 & 8.191 & 0.000 \\ 0.386 & 8.191 & 82.121 & 1.336 \\ 0.268 & 0.000 & 1.336 & 134.757 \end{pmatrix} \quad . \tag{37}$$

The members of the primary diagonal can be translated into effective thermal capacity Ξ_i according to equation (32) by simply dividing by the frequency of rotation ω. For the daily period, ω has the value $\frac{\pi}{12}$ h^{-1} (0.2618 h^{-1}). The subsequent value of Ξ_0, corresponding to the exterior, is thus $\frac{72.169}{\omega} = 275.7$ Whm^{-1}K^{-1}; this, however, is of no practical significance.

The quantity 55.785 Wm^{-1}K^{-1} in matrix (37) implies that a heating load amplitude (length-related) 55.785 Wm^{-1} in room 1 induces a temperature amplitude of 1 K in the same if the temperature is held constant (i.e. the temperature amplitude vanishes) in all other rooms — including the exterior environment. The effective thermal capacity Ξ_1 of room 1 has the value $\frac{55.785}{\omega} = 213.1$ Whm^{-1}K^{-1}.

With $\Xi_2 = \frac{82.121}{\omega} = 313.7$ Whm^{-1}K^{-1}, room 2 posesses a considerably larger thermal capacity than room 1. Room 3 has the largest thermal capacity of $\Xi_3 = \frac{134.757}{\omega} = 514.7$Whm^{-1}K^{-1}. These results appear quite plausible looking at illustration 2.

The quantities off of the diagonal in matrix (37) also allow a simple, coherent interpretation. The reciprocal effect between rooms 1 and 2 shall be singled out to illustrate this. Here the absolute value is 8.191 Wm^{-1}K^{-1} for the generalized conductance with the indices 1 and 2. A glance at equation (15) immediately makes clear the significance of this conductance. Since $i = 1$, the integral of the heat flow density is taken over the boundary of room 1, thus determing the heat flow amplitude in room 1. Because $j = 2$, the basic solution associated with room 2, which assumes a temperature amplitude of 1 in this and only this room, is implemented; all other temperature amplitudes vanish. Thus the declared value of 8.191 Wm^{-1}K^{-1} implies that an oscillating heat flow passes into and out of room 1 respectively if the temperature in room 2 fluctuates with an amplitude of 1 K.

The reciprocal effect of temperature fluctuations occurs most strongly between the interior spaces 1 and 2; as expected, the reciprocation between rooms 1 and 3 is negligable.

Fluctuations of the exterior temperature affect the individual interior spaces differently: room 1 is coupled with 18.892 Wm^{-1}K^{-1}, whereas all the other rooms are coupled to the exterior very weakly by comparison.

This interpretation can easily be extended over matrix $\mathcal{L}_{24}$ of the complex generalized conductance values for a period of 24 hours, which supplies the phase shifts between heat flows and temperatures in addition to the previously discussed absolute values.

A deduction of the effective thermal capacities Ξ_i^* out of the generalized conductance matrix (36) is left to the reader. Hereby one also realizes that there is little sense in calculating the effective thermal capacity for room 1 because of the strong coupling to the exterior. In a practical application, this situation would be further intensified due to the effects of glazing.

6 Review and prospect

A correct treatment of heat storage processes in building structures is only possible by means of a thorough examination of the relevant time–dependant heat flows. The concept presented here is only seemingly restricted to periodically fluctuating processes since a generalization which incorporates aperiodic functions as well is principally possible with the use of Laplace transforms [15].

The segment of this discourse most relevant to the practical application of evaluating a building's thermal performance is the generalization of the already familiar conduction value concept for steady–state conditions to include periodically time–dependant conditions as well. This provides the basis for calculating the thermal performance of entire buildings in which two– and three–dimensional heat conduction is considered — without a serious increase in complications as compared to one–dimensional, time–dependant calculations.

The interpretation of effective thermal capacity serves to illuminate otherwise analyzed principles and should not be taken as a recommendation for the use of this concept in researching thermal properties of building materials and assemblies.

The effects of heat sources, also important for practical applications, has not been treated here; they shall be examined in an independent discourse.

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